

# Assignment 1

(27 January, 2026)

Submission Deadline: 07 February, 2026

Course: Algebraic Topology II (KSM4E02)

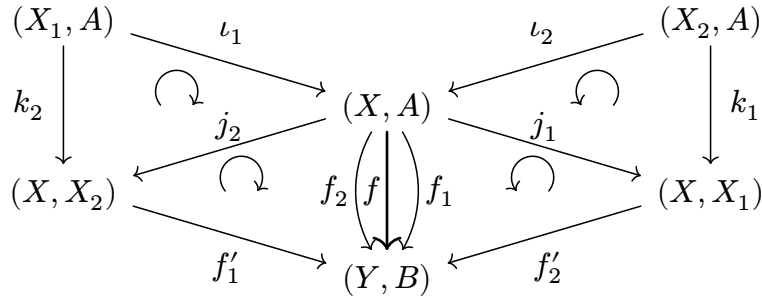
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**Q1. (Proper Triad and Summation of Maps)** Let  $(X; X_1, X_2)$  be a proper triad with  $X = X_1 \cup X_2$ , and set  $A := X_1 \cap X_2$ . Let  $f, f_1, f_2 : (X, A) \rightarrow (Y, B)$  be maps such that

$$f_1|_{X_1} = f|_{X_1}, \quad f_1(X_2) \subset B \qquad f_2|_{X_2} = f|_{X_2}, \quad f_2(X_1) \subset B.$$

Show that  $f_* = f_{1*} + f_{2*}$ .

**Hint :** Consider the space-level diagram



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**Q2.** Let  $(X; X_1, X_2)$  be a proper triad. Verify that  $(X_1 \cup X_2; X_1, X_2)$  is again a proper triad. Show that the diagram

$$\begin{array}{ccc} H_{n+1}(X, X_1 \cup X_2) & \xrightarrow{\partial} & H_n(X_1, X_1 \cap X_2) \\ \partial_1 \downarrow & & \downarrow \partial_2 \\ H_n(X_1 \cup X_2) & \xrightarrow{\Delta} & H_{n-1}(X_1 \cap X_2) \end{array}$$

commutes, where  $\partial$  is the boundary map of the long exact sequence of the triad  $(X; X_1, X_2)$ , and  $\Delta$  is the boundary map of the Mayer-Vietoris sequence of  $(X_1 \cup X_2; X_1, X_2)$ . The boundary maps  $\partial_i$  are from the long exact sequence of the corresponding pairs.

3 + 7 = 10

**Q3.** Let  $(X; X_1, X_2)$  be a proper triad. Consider the diagram

$$\begin{array}{ccc} H_{n+1}(X, X_1 \cup X_2) & \xrightarrow{\partial_1} & H_n(X_1, X_1 \cap X_2) \\ \partial_2 \downarrow & & \downarrow \partial \\ H_n(X_2, X_1 \cap X_2) & \xrightarrow{\partial} & H_{n-2}(X_1 \cap X_2) \end{array}$$

where  $\partial_i$  are the boundary maps of the corresponding triad, and the two  $\partial$ 's are the boundary maps of the corresponding pairs. Show that the diagram *anticommutes*, i.e, show that  $\partial \circ \partial_1 + \partial \circ \partial_2 = 0$ .

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**Q4.** Prove the relative Mayer-Vietoris sequence.

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